Is Democracy Possible?

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What are we talking about?

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- More generally, we're talking about a specific form of group decision making —
  - Deciding whether a building project should take place
  - Deciding whether an amendment to a law should pass
  - Choosing what/where to eat with a group of friends

What are we trying not to talk about?

- Why democracy is a good/bad idea

The process

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So what can go wrong?

- voting fraud - carousel voting, intimidation
  - statistical methods can sometimes be used to detect anomalies.
- counting fraud - particularly in automated voting machines
  - Verifying that the voting program works as desired; having source code is not enough.
  - Verifying the integrity of the data; encryption is not enough
  - If someone has physical access to the voting machine, it's virtually impossible to secure.

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  - If someone has physical access to the voting machine, it's virtually impossible to secure.
- But what about the voting system itself?

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- Ensure "good" decisions are made

> *Democracy is the recurrent suspicion that more than half of the people are right more than half the time.*

– E.B. White

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What is the point of democracy?

- ~~Ensure "good" decisions are made~~
- Reflect the will of the people
  - Which people? All of them?
  - What if 51% of people really don't like the other 49%?

Modelling the problem

- The purpose of voting is to obtain a *collective* preference (or *social choice*) from a set of individual preferences.
- A preference is some sort of "goodness" ordering over outcomes

$$pizza >_{nir} curry >_{nir} stir\ fry$$

$$pizza >_{frank} stir\ fry >_{frank} curry$$

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$$pizza >_{frank} stir\ fry >_{frank} curry$$

$$pizza > stir\ fry = curry$$

FPTP

- 7 people are trying to decide whether to eat Pizza or Chinese.
  - 3 voters $P > C > I$
  - 2 voters $C > P > I$
  - 2 voters $I > C > P$
- Chinese will win with 4 votes to 3.

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- Chinese will win with 4 votes to 3.
- If the choice of indian is introduced, then pizza will win and chinese will come second.
- We've introduced an "irrelevant" alternative (as it still comes last) which has reversed the outcome.
- This feels "unfair"

Properties of Voting Systems

The following properties of voting systems are generally considered desirable:

- U : Anyone can have any sort of consistent preference — anyone can vote for anything. This is known as the condition of *universal domain*.
- P : If everyone voting prefers X to Y, then in the result, X should be ranked more highly than Y. This is the *weak Pareto principle*.
- D : There is no individual such that no matter what anyone else prefers, they can decide on the outcome. This is the *non-dictatorship principle*.

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- I : If a voting system combines two objects a, b so that $a \geq b$ for a set of individuals who have different orderings (e.g. $a \geq_1 b, b \geq_2 a, b \geq_3 a$), then as long as these different orderings hold, the voting system will always result in $a \geq b$.
- In other words, a's relation to c (and c's to b) doesn't matter.

Example

$$a \geq b \qquad if \qquad (acbd, dbac)$$

Then

$$(abcd, bdca)$$
$$(abcd, bacd)$$
$$(acdb, bcda)$$

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Can we find a voting system that satisfies all of these properties?

NO!

So why do we care?

Given a finite number of individuals (even 2!), and at least three possibilities, there is no way to create a voting system for which conditions U, P, D and I hold.

Proof

Let's assume we have n people voting over possibilities $a, b, c, \ldots$.

- Let's assume that for all individuals rank a the highest, and b the lowest.
- Since a is preferred over every other outcome, by P it must be ranked most highly.
- Similarly, b is ranked as the least preferred outcome.

R_1	$\dots$	R_{m-1}	R_m	R_{m+1}	$\dots$	R_n	*outcome*
a	$\dots$	a	a	a	$\dots$	a	a
.	$\dots$	.	.	.	$\dots$	.	.
b	$\dots$	b	b	b	$\dots$	b	b

R_1	...	R_{m-1}	R_m	R_{m+1}	...	R_n	*outcome*
a	...	a	a	a	...	a	a
.	...	.	.	.	...	.	.
b	...	b	b	b	...	b	b

Now let's lift b up for R_1 by 1 position

R_1	$\dots$	R_{m-1}	R_m	R_{m+1}	$\dots$	R_n	*outcome*
a	$\dots$	a	a	a	$\dots$	a	a
.	$\dots$	.	.	.	$\dots$	.	.
b	$\dots$	.	.	.	$\dots$	.	.
.	$\dots$	b	b	b	$\dots$	b	.

R_1	...	R_{m-1}	R_m	R_{m+1}	...	R_n	*outcome*
a	...	a	a	a	...	a	a
.	...	.	.	.	...	.	.
b	...	.	.	.	...	.	.
.	...	b	b	b	...	b	.

Repeat until b is R_1's most preferred outcome.

R_1	$\dots$	R_{m-1}	R_m	R_{m+1}	$\dots$	R_n	*outcome*
b	$\dots$	a	a	a	$\dots$	a	a
a	$\dots$	.	.	.	$\dots$	.	.
.	$\dots$	.	.	.	$\dots$	.	.
.	$\dots$	b	b	b	$\dots$	b	.

R_1	...	R_{m-1}	R_m	R_{m+1}	...	R_n	*outcome*
b	...	a	a	a	...	a	a
a	...	.	.	.	...	.	.
.	...	.	.	.	...	.	.
.	...	b	b	b	...	b	.

- Now since we've only actually reordered b and a, by I, a must be first or second in the outcomes.
- Let's assume it remains at the top.
- So we repeatedly raise b for the 2nd person, 3rd person etc, until b gets to the top.
- Let's say this happens for person m
- Note that if we end up doing this for all R's, by P we're guaranteed to have b as the most preferred outcome, so this is always possible.

R_1	...	R_{m-1}	R_m	R_{m+1}	...	R_n	*outcome*
b	...	b	a	a	...	a	a
a	...	a	b	.	...	.	.
.	...	.	.	.	...	.	.
.	...	.	.	b	...	b	.

Again, since we're only dealing with a and b, by I this is the only outcome that should be affected.

R_1	...	R_{m-1}	R_m	R_{m+1}	...	R_n	outcome
b	...	b	b	a	...	a	b
a	...	a	a	.	...	.	a
.	...	.	.	.	...	.	.
.	...	.	.	b	...	b	.

Again, since we're only dealing with a and b, by I this is the only outcome that should be affected.

R_1	$\ldots$	R_{m-1}	R_m	R_{m+1}	$\ldots$	R_n	*outcome*
b	$\ldots$	b	a	a	$\ldots$	a	a
a	$\ldots$	a	b	.	$\ldots$	.	.
.	$\ldots$	.	.	.	$\ldots$	.	.
.	$\ldots$	.	.	b	$\ldots$	b	.

R_1	$\ldots$	R_{m-1}	R_m	R_{m+1}	$\ldots$	R_n	*outcome*
b	$\ldots$	b	b	a	$\ldots$	a	b
a	$\ldots$	a	a	.	$\ldots$	.	a
.	$\ldots$	.	.	.	$\ldots$	.	.
.	$\ldots$	.	.	b	$\ldots$	b	.

- Let's move a to the bottom for all $i < m$ and to the 2nd most preferred position for all $i > m$.

R_1	...	R_{m-1}	R_m	R_{m+1}	...	R_n	*outcome*
b	...	b	a	.	...	.	.
.	...	.	b	.	...	.	.
.	...	.	.	a	...	a	.
a	...	a	.	b	...	b	.

R_1	...	R_{m-1}	R_m	R_{m+1}	...	R_n	*outcome*
b	...	b	b	.	...	.	b
.	...	.	a	.	...	.	.
.	...	.	.	a	...	a	.
a	...	a	.	b	...	b	.

- Let's move a to the bottom for all $i < m$ and to the 2nd most preferred position for all $i > m$.
- For the highlighted case, b hasn't moved with regards to anything else and must therefore be ranked most highly due to I.
- Since b was only exchanged with a in the highlighted case, it cannot change ranking with anything other than a. So in the first situation, b must rank highest apart from possibly a.

Back to case 1

R_1	$\ldots$	R_{m-1}	R_m	R_{m+1}	$\ldots$	R_n	*outcome*
b	$\ldots$	b	a	a	$\ldots$	a	a
a	$\ldots$	a	b	.	$\ldots$	.	.
.	$\ldots$	.	.	.	$\ldots$	.	.
.	$\ldots$	.	.	b	$\ldots$	b	.

R_1	$\ldots$	R_{m-1}	R_m	R_{m+1}	$\ldots$	R_n	*outcome*
b	$\ldots$	b	a	.	$\ldots$	.	.
.	$\ldots$	.	b	.	$\ldots$	.	.
.	$\ldots$	.	.	a	$\ldots$	a	.
a	$\ldots$	a	.	b	$\ldots$	b	.

- So we know that in the case at the bottom, b must rank highest apart from possibly a.
- Comparing, note that a and b haven't moved w.r.t each other.
- So since b must rank highest in the bottom case apart form a, a must rank highest in the bottom case.

Back to case 1

R_1	$\dots$	R_{m-1}	R_m	R_{m+1}	$\dots$	R_n	*outcome*
b	$\dots$	b	a	a	$\dots$	a	a
a	$\dots$	a	b	.	$\dots$	.	.
.	$\dots$	.	.	.	$\dots$	.	.
.	$\dots$	.	.	b	$\dots$	b	.

R_1	$\dots$	R_{m-1}	R_m	R_{m+1}	$\dots$	R_n	*outcome*
b	$\dots$	b	a	.	$\dots$	.	a
.	$\dots$	.	b	.	$\dots$	.	.
.	$\dots$	.	.	a	$\dots$	a	.
a	$\dots$	a	.	b	$\dots$	b	.

- So we know that in the case at the bottom, b must rank highest apart from possibly a.
- Comparing, note that a and b haven't moved w.r.t each other.
- So since b must rank highest in the bottom case apart form a, a must rank highest in the bottom case.

So What?

We've shown that if a is ranked lowest for $i < m$ and second lowest for $m > i$ and highest for $i = m$, a will be highest in the vote.

R_1	...	R_{m-1}	R_m	R_{m+1}	...	R_n	*outcome*
.	...	.	a	.	...	.	a
c	...	c	c	c	...	c	.
b	...	b	b	a	...	a	.
a	...	a	.	b	...	b	.

- Let's switch the rankings of a and b for $i > m$.
- Can b move above a in the outcomes?

R_1	$\dots$	R_{m-1}	R_m	R_{m+1}	$\dots$	R_n	*outcome*
.	$\dots$	.	a	.	$\dots$	.	a
c	$\dots$	c	c	c	$\dots$	c	.
b	$\dots$	b	b	b	$\dots$	b	.
a	$\dots$	a	.	a	$\dots$	a	.

- Let's switch the rankings of a and b for $i > m$.
- Can b move above a in the outcomes?
- No as $c > b$ so by P c has to rank above b.
- Therefore a remains at the top, and c ranks above b.

Final Step!

R_1	...	R_{m-1}	R_m	R_{m+1}	...	R_n	*outcome*
c	...	c	a	c	...	c	a
.	...	.	c	.	...	.	.
b	...	b	b	b	...	b	c
a	...	a	.	a	...	a	b

- Create an arbitrary set of profiles, except for R_m for who $a > b$.
- I means that c can't have an effect on the rankings of a and b.
- The rankings between a and c are as in the previous step (i.e. $c > a$ except for R_m) — by I a must remain preferred over c.
- c is above b so by P it is preferred.
- So $a > c$ and $c > b$ so $a > b$ whenever $a >_{R_m} b$
- In other words, R_m is a dictator for choice a.

But...

- Could we have different dictators for different choices (e.g. one for a, a different one for b etc)?

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- Could we have different dictators for different choices (e.g. one for a, a different one for b etc)?
- No; as what would happen when both dictators try exert their power?
- We have used I, P and U to show that D cannot hold.
- No voting system can satisfy all of the desired conditions simultaneously!

Is it all bad news?

- So no voting system is perfect.
- But we could lift one of the requirements.

Lifting U

- In some situations, it is possible to constrain the types of preferences individuals can have.
- For example, selecting the volume of music for a party.
- It's been shown that in such situations, majority rule voting works.

Lifting P

- Not requiring P is not as useful; it has been shown that either a dictator still exists, or an *inverse dictator*.
- For an inverse dictator, if $a >_i b$ then $b > a$.

Lifting I

- If we lift I, then as seen in FPTP, voting for an "irrelevant" alternative can affect the outcome.
- This means that a voter could change the winner by voting for someone that they do not really want to vote for.
- In other words, *strategic voting* is a necessary feature of any voting system which ignores I. This include FPTP, AV, Borda and most other "widely used" voting systems.

Lifting I

- Strategic voting means a voter must consider all the other voter's choices when making their choice.
- "If a votes x then I should vote y. But if a thinks I'll vote y, they'll vote z, in which case I should vote x, ..."
- Voting becomes a *game theoretic* problem.
- Solving game theoretic problems can be hard:
  - Strategic voting could mean an unexpected (and unwanted) outcome.
  - But computing an optimal voting strategy could be very difficult, disincentivising such behaviour.

And another thing...

- Note that we only spoke about 3 or more alternatives.
- What if we've only got 2? Then Arrow's theorem doesn't hold.
- So we could vote on 2 issues.
- Why not always limit to 2 alternatives (e.g. if there are 4 alternatives, pit 2 of them against each other in two "preliminary rounds") and then have the winners fight it out?
- The order in which the alternatives are given alters the final outcome.

So where are we?

- The voting process is vulnerable at various points
  - Social, political and technical vulnerabilities occur when running elections.
  - Mathematical vulnerabilities appear when trying to create a fair voting mechanism.
- The latter result indicates that strategic voting is always possible.
- But what if, instead of trying to find a perfect voting mechanism, voters could change their preferences?
  - Perhaps access to better explanations about outcomes of decisions could align people's preferences?
  - If so, increasing debate, participative democracy etc, might be the best way to make democracy work.